The Study Contact Bending Stress Two-Layer Plates from Fiberglass with Interfacial Defects of Structure

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Submission: September 09, 2017; **Published:** October 16, 2017

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Abstract

The presented version of the nonlinear discrete-structural theory adequately reflects the work of real structures. Obtained are in satisfactory compliance with the theoretical and experimental data. The magnitude of the transverse shear stresses of contact increases 4-5 times in a zone bearing the circuit, which leads to interfacial failure of the adhesive layer. Thus, the proposed model allows the calculation to locate the contact area, touch, pressure, and the changing nature of the state stress at the interface.

Keywords: Structure; Bending stress; Fiberglass; Anisotropic; Shell

Introduction

The significant difference between mechanical and physicalproperties of the individual components of the structure of layered thin-walled structural elements have driven to the creation of adequate discrete-structural theories to calculate this group of problems. Contact problem of mechanics of multilayered plates and shells is considered in [1-6], which, based on the discrete approach built functional and obtained a system of equations for solving such problems, provided non-ideal contact layers. A method for solving nonlinear problems of contact between the two shells of different shapes and equidistant layers can be found in [7-10], on the other handdetailed analysis of the latest results and trends in the development of the discrete-structural theory of laminated plates and shells found in [11].

In this paper, we propose a variant of the geometrically nonlinear dispersion specifically-layered structure theory of structural elements and study nature of the change of contact stresses in bending double-layer plates fiberglass with interfacial defects of the material structure. The derivation equilibrium equations and geometric ratio take into account the influence of transverse shear deformation and compression. The results of theoretical studies are compared with experimental data.

Formulation of Analytical Model

According to the theory of discrete structural, mathematical model considered, the multilayer shell consists of n thin anisotropic layers. The volume of (n) rigid layers is $V = \sum_{i=1}^{n} V_i$. Each cladding layer undeformed assigned to an orthogonal curvilinear coordinate system, α^i (i=1,2), $z^{(k)}$.Coordinate directed along the common normal $\vec{m}^{(k)}$ to the middle surface $S^{(k)}$ and equidistant surface $S_z^{(k)}$ k-th layer. Index "z" with the introduction of other characters means that the corresponding values are to the point $(\alpha^1,\alpha^2,z^{(k)})$equidistant surface $S_z^{(k)}$.

Total displacement vector $\vec{u}_z^{(k)}$ k-th point of the hard layer according to the refined Tomachinko shell theory can be represented as:

$$\vec{u}_z^{(k)} = \vec{u}^{(k)} + z^{(k)}\vec{\gamma}^{(k)} + \varphi^{(k)}(z)\vec{\psi}^{(k)}, \quad (1)$$

Where $\vec{u}^{(k)}$ – displacement vector points of the middle surface $S^{(k)}$ $\vec{\gamma}^{(k)}$ –Vector function of the angles of rotation and compression of the fibers in the direction normal to the undeformed middle surface $S^{(k)}$ during deformation. $\varphi^{(k)}(z)$–Nonlinear continuous distribution function tangential displacements through the thickness of the k-th layer, analysis and approximation given in [11]

$$\vec{\psi}^{(k)}(\alpha_1^{(k)},\alpha_2^{(k)}) \quad -$$

Vector shift function. Covariant components of vectors $\vec{u}^{(k)},\vec{\gamma}^{(k)},\vec{\psi}^{(k)}$ recorded using the following expressions:

$$\vec{u}^{(k)} = \vec{r}^{(k)i}u_i^{(k)} + \vec{m}^{(k)}w^{(k)}, \quad \vec{\gamma}^{(k)} = \vec{r}^{(k)i}\gamma_i^{(k)} + \vec{m}^{(k)}\gamma^{(k)},$$

$$\vec{\psi}^{(k)} = \vec{r}^{(k)i}\psi_i^{(k)}. \qquad (2)$$

Deformation tensor components $(\alpha^1, \alpha^2, z^{(k)})$ defined as half the corresponding metric tensor components before and after deformation:

$$2\varepsilon_{ij}^{(k)z} = g_{ij}^{(k)*} - g_{ij}^{(k)}, \quad 2\varepsilon_{i3}^{(k)z} = g_{i3}^{(k)*} - g_{i3}^{(k)}, \quad 2\varepsilon_{33}^{(k)z} = g_{33}^{(k)*} - 1. \,(3)$$

The finals of the equilibrium equations and boundary conditions for the solution of contact problems of laminated shells are usually made on the basis of Reissner variational principle. Considering that in the direction normal to the middle surface of the individual layers of the shell axial line of general and topical coordinate systems are combined and also bias with the topical coordinate sursurfaces of the middle surface Layers, the variational equation for the Reissner principle multilayer shell write in the following format:

$$\delta R = \sum_{k=1}^{n} \delta R^{(k)} = \sum_{k=1}^{n} \delta A_R^{(k)} - \sum_{k=1}^{n} \iiint_{V^{(k)}} \delta\left(\sigma_{(k)}^{\alpha\beta}\varepsilon_{\alpha\beta}^{(k)} - F^{(k)}\right) dV = 0 \quad (\alpha,\beta=1,2,3). \,(4)$$

Punctuation starts from the layers of negative values of z from 1 to n. Wherein $F^{(k)}$ - specific additional work strain k-th layer, $\sigma_{(k)}^{\alpha\beta}, \quad \varepsilon_{\alpha\beta}^{(k)}$ —components of the stress and the strain tensors.

If we assume that the dual sursurfaces of the k-th layer of the conditions of full contact:

$$u_\beta^{(k,k-1)} = u_\beta^{(k-1,k)}, \quad X_{(k,k-1)}^\beta = X_{(k-1,k)}^\beta \,(5)$$

Qr in vector form:

$$\vec{u}_{\mp}^{(k)}\left(\alpha_i^{(k)}, \ -h^{(k)}/2\right) = \vec{u}_{\mp}^{(k-1)}(\alpha_i^{(k-1)}, h^{(k-1)}/2),$$

$$\vec{X}_{(k)}\left(\alpha_i^{(k)}, \ -h^{(k)}/2\right) = \vec{X}_{(k-1)}\left(\alpha_i^{(k-1)}, \ h^{(k-1)}/2\right) \quad (i=1,2), \,(6)$$

The difference of the elementary work of external forces $\delta\grave{A}_R$ represented by the following formula:

$$\grave{A}_R = \sum_{k=1}^{n} \delta A_R^{(k)} = \sum_{k=1}^{n} \iint_{S_{(k)}} \left(\vec{X}_{(k)}\delta\vec{u}^{(k)} + M_{(k)}^i\vec{r}_i^{(k)}\delta\vec{\gamma}^{(k)} + B_{(k)}^i\vec{r}_i^{(k)}\delta\vec{\psi}^{(k)} + M_{(k)}^3\delta\varepsilon_{33}^{(k)z}\right)dS +$$

$$+\sum_{k=1}^{n}\int_{l_1^{(k)}} \left(\bar{O}_{(k)}^S\delta\vec{u}^{(k)} + \vec{G}_{(k)}^S\delta\vec{\gamma}^{(k)} + \vec{L}_{(k)}^S\delta\vec{\psi}^{(k)}\right)dl +$$

$$+\sum_{k=1}^{n}\int_{l_2^{(k)}} \left(\bar{O}_{(k)}\delta\vec{u}^{(k)} + G_{(k)}\delta\vec{\gamma}^{(k)} + L_{(k)}\delta\vec{\psi}^{(k)} + \left(\vec{u}^{(k)} - \vec{u}_S^{(k)}\right)\delta\bar{O}_{(k)} + \right.$$

$$\left. + \left(\vec{\gamma}^{(k)} - \vec{\gamma}_S^{(k)}\right)\delta\vec{G}_{(k)} + \left(\vec{\psi}^{(k)} - \vec{\psi}_S^{(k)}\right)\delta\vec{L}_{(k)}\right)dl \,. \,(7)$$

Where $S_{(k)}$ - middle surface of the k-th layer; $l_1^{(k)}, \ l_2^{(k)}$ - part of the loop , $l^{(k)}$. Vectors external forces $\vec{O}_{(k)}$, moments $\vec{M}_{(k)}$ and additional points $\vec{B}_{(k)}$, are inclusive in the equation (7), are defined by:

$$\vec{X}_{(k)} = \vec{X}_{(k)}^+ - \vec{X}_{(k)}^- + \int_{-h^{(k)}/2}^{h^{(k)}/2} \vec{P}^{(k)}d\mp , \quad \vec{M}_{(k)} = \frac{h^{(k)}}{2}\left(\vec{X}_{(k)}^+ - \vec{X}_{(k)}^-\right) + \int_{-h^{(k)}/2}^{h^{(k)}/2} \vec{P}^{(k)}\mp^{(k)}d\mp$$

$$\vec{B}_{(k)} = \varphi^{(k)}\left(\frac{h^{(k)}}{2}\right)\left(\vec{X}_{(k)}^+ - \vec{X}_{(k)}^-\right) + \int_{-h^{(k)}/2}^{h^{(k)}/2} \vec{P}^{(k)}\varphi^{(k)}(z)d\mp \qquad (8)$$

Where the vectors $\vec{X}_{(k)}^+, \quad \vec{X}_{(k)}^-$ include the contravariant components of the tensor of contact stresses :

$$\sigma_{(k)}^{i3+}, \quad \sigma_{(k)}^{i3-} \quad (i=1,2,3)$$

$$\vec{O}_{(k)}^+ = \sigma_{(k)}^{i3+}\vec{\rho}_i^{(k)*} + \sigma_{(k)}^{33+}\vec{m}^{(k)*} , \quad \vec{O}_{(k)}^- = \sigma_{(k)}^{i3-}\vec{\rho}_i^{(k)*} + \sigma_{(k)}^{33-}\vec{m}^{(k)*} \quad (i=1,2) \,(9)$$

Subscripts "+" and "- " indicate the top and bottom surfaces of the k-th layer. A similar entry are vectors of the external load $\vec{q}_{(n)}^+, \quad \vec{q}_{(1)}^-$:

$$\vec{q}_{(n)}^+ = q_{(n)}^{i3+}\vec{\rho}_i^{(n)*} + q_{(n)}^{33+}\vec{m}^{(n)*}, \quad \vec{q}_{(1)}^- = q_{(1)}^{i3-}\vec{\rho}_i^{(l)*} + q_{(1)}^{33-}\vec{m}^{(l)*} \quad (i=1,2). \,(10)$$

Vector $\vec{P}^{(k)}$ takes into account the effect of its own weight. contravariant components $M_{(k)}^i$, $M_{(k)}^3$ vector moment $\vec{M}_{(k)}$ with respect to the basis vectors $\vec{r}_i^{(k)*}$ and $\vec{m}^{(k)*}$ as to the equation:

$$\vec{M}_{(k)} = M_{(k)}^i\vec{r}_i^{(k)*} + \vec{m}^{(k)*}M_{(k)}^3 \,(11)$$

Also, elementary work (7) the layer of the shell is differentiated mainly by the vector $\vec{\Phi}_{(k)}^S$, the main point $\bar{G}_{(k)}^S$, additional main point $\bar{L}_{(k)}^S$, that arise from the action of the external forces on the contour $l_1^{(k)}$, main vector $\vec{\Phi}_{(k)}$, the main point $\vec{G}_{(k)}$, additional main point $\bar{L}_{(k)}$, associated with the tension at loop $l_2^{(k)}$ by virtue of a predetermined displacement of the contour points $\vec{u}_S^{(k)}$.

The second term of equation (4) should be represented in the form:

$$\delta\Pi_R = \sum_{k=1}^{n}\left(\delta\Pi_{1R}^{(k)} + \delta\Pi_{2R}^{(k)}\right) = \sum_{k=1}^{n}\iiint_{V^{(k)}}\sigma^{\alpha\beta}\delta\eta_{\alpha\beta}^{(k)}dV - \sum_{k=1}^{n}\iiint_{V^{(k)}}\left(\partial F^{(k)}/\partial\sigma_{(k)}^{\alpha\beta} - \eta_{\alpha\beta}^{(k)}\right)\delta\sigma_{(k)}^{\alpha\beta}dV , \qquad (12)$$

Where

$$\delta\Pi_R = \sum_{k=1}^{n}\left(\delta\Pi_{1R}^{(k)} + \delta\Pi_{2R}^{(k)}\right) = \sum_{k=1}^{n}\iiint_{V^{(k)}}\sigma^{\alpha\beta}\delta\eta_{\alpha\beta}^{(k)}dV - \sum_{k=1}^{n}\iiint_{V^{(k)}}\left(\partial F^{(k)}/\partial\sigma_{(k)}^{\alpha\beta} - \eta_{\alpha\beta}^{(k)}\right)\delta\sigma_{(k)}^{\alpha\beta}dV ,$$

$$\delta\Pi_{2R}^{(k)} = -\iiint_{V^{(k)}}\delta W_{(k)}^f dV = -\iiint_{V^{(k)}}\left\{\left(\partial F^{(k)}/\partial\sigma_{(k)}^{ij} - \varepsilon_{ij}^{(k)\ne}\right)\delta\sigma_{(k)}^{ij} + \left(\partial F^{(k)}/\partial\sigma_{(k)}^{i3} - 2\varepsilon_{i3}^{(k)\ne}\right)\times \right.$$

$$\left. \times\delta\sigma_{(k)}^{i3} + \left(\partial F^{(k)}/\partial\sigma_{(k)}^{33} - \varepsilon_{33}^{(k)\ne}\right)\delta\sigma_{(k)}^{33} \right\}dV \quad (i,j=1,2)$$

And offset equations (3) (4), (7), (12) and varying the mutually independent displacements and stresses can be obtained for each individual cladding layer system of equilibrium equations, physical relationships, static and kinematic boundary conditions. Application of the generalized Hooke's law, the nonlinear theory of the average bending diaphragm [12] simplifies the derivation of the equilibrium equations and boundary conditions. The transition to the physical components used in this study tensors, the inference of the equilibrium equations and boundary conditions can be found in [13].

For a rotating shell which comprises n layers of coaxial sursurfaces of revolution, the solution of partial differential equations is as follows:

$$\frac{\partial\vec{Y}^{(k)}}{A_{(k)}\partial\alpha_1^{(k)}} = D_o^{(k)}\vec{Y}^{(k)} + D_1^{(k)}\frac{\partial\vec{Y}^{(k)}}{B_{(k)}\partial\alpha_2^{(k)}} + D_2^{(k)}\frac{\partial^2\vec{Y}^{(k)}}{B_{(k)}^2\partial\alpha_2^{(k)2}} + \vec{f}^{(k)}, \quad k=1,2...n$$

Where

$$\vec{Y}^{(k)} = \left\{\vec{I}_{11}^{(k)}, T_{12}^{(k)}, LQ_1^{(k)}u, M_{11}^{(k)}u \quad \psi_2^{(k)}, \ _{11}^{(k)}, \ _{12}^{(k)}, \ _1^{(k)}, \quad _2^{(k)}, \ ^{(k)}, \gamma_1^{(k)}, \gamma_2^{(k)}, \psi_1^{(k)}, \psi_2^{(k)}\right\}^T,$$

$$\vec{f}^{(k)} = \left\{f_1^{(k)}, f_2^{(k)}, ..., f_{14}^{(k)}\right\}, \qquad (13)$$

$D_0^{(k)}, D_1^{(k)}, D_2^{(k)}$ - Square matrices *14th.* The main unknown function occupies a value that define the boundary conditions

on the lateral contour of the **k-th** layer denotedshell. Due to the lack ofspace to show resolution systemequations, physical and geometrical ratio in developed formis notpossible.Kinematic and static contact condition (5) facial surface**k-th** layer and associated sursurfaces,**k + 1** and **k - 1-th** layer, according to the notation introduced earlier take this equation:

$$2u_i^{(k+1)} + u_i^{(k+1)} - \frac{h^{(k+1)}}{2}\gamma_i^{(k+1)} + \frac{h^{(k+1)}}{2}\gamma_i^{(k+1)} - \varphi^{(k+1)}\left(\frac{h^{(k+1)}}{2}\right)\psi^{(k+1)} + \varphi^{(k+1)}\left(\frac{h^{(k-1)}}{2}\right)\psi^{(k+1)}$$

$$(i = 1,2);\ 2w^{(k)} = w^{(k+1)} + w^{(k-1)} - \frac{h^{(k+1)}}{2}\gamma^{(k+1)} + \frac{h^{(k-1)}}{2}\gamma^{(k-1)} \quad (14)$$

$$\sigma_{i3}^{(k)+} = \sigma_{i3}^{(k+1)-}, \sigma_{i3}^{(k)-} = \sigma_{i3}^{(k-1)+},\ (i = 1,2);$$

$$\sigma_{33}^{(k)+} = \sigma_{33}^{(k+1)-}, \sigma_{33}^{(k)+} = \sigma_{33}^{(k+1)-}. \quad (15)$$

Presented the kinematic relations (14) in the construction of resolving a system of equations (13) for the entire package element layers and proceed static conditions on personal contact mating sursurfaces (15) on the basis of the penalty function method [11], we can create an algorithm for solving the contact problem discrete structural theory of multilayer shells. If between *k-th* and *k + 1-th* cladding layers. Assuming no kinematic connections at the surface interface of these layers may occur effort unknown vectors $\vec{q}_{(k)}$, $\vec{q}_{(k+1)}$ contact interaction. According to the 3rd Newton's second law is a dependence $\vec{q}_{(k)} = -\vec{q}_{(k+1)}$ To account for the impact of the efforts of the contact interaction of the layers in the variational equation Reissner principle (4) it is necessary to introduce a term that takes into account the work force of contact interaction in the motion vector of each layer portion of the mating surface:

$$A_q = \sum_{m=k}^{k+1} \iint_{S_z^{(k,k-1)}} \vec{q}_{(m)}\vec{u}_z^{(m)}dS. \quad (16)$$

Efforts contact interaction $\vec{q}_{(k)} = q_{(k)}^i\vec{r}_i^{(k)} + q_{(k)}^3\vec{m}^{(k)}$ arise when the condition:

$$(\vec{u}_z^{(k)} - \vec{u}_z^{(k+1)}) < 0 \quad (17)$$

Coupling in areas harsh layers, in the case, where the inequality (17) is not performed while moving the field of points $S_z^{(k,k+1)}$ during deformation, the contact pressure $\vec{q}_{(k)}$ in equation (13) takes the value $\vec{q}_{(k)} = 0$. Solving the system of equations (13), it is easy to find the value of a given accuracy of the contact pressure on the basis of the iterative method suggested in [12].

Results and Discussion

Test cases investigated plate round shape in terms of diameter 0.16m regular structure of fiberglass. Considered plates were made of 4 - layers of glass TG 430 - C (100) (producer - Latvia). The binder used polyester orthophthalic resin with low styrene emission Cristic 2 - 446PA (producer - UK). Physical and mechanical properties of fiberglass plates determinedmined as follows. Initially, according to GOST 25,601- 80, determined by the elastic modulus and Poisson's ratio when stretchingthe sample is made of fiberglass. Therefore mechanical testingmakes it possible to assert that the records can be considered material

class-classify as a transversely isotropic ($E11 = E22 = 1.5 \cdot 104$ *MPa,v12 = v21 = 0.12*). The remainder of the physical and mechanical characteristics of black glass-stick determined by the integral over the entire packet layers of the plate on the basis ofdependencies work [12].

when the elastic moduli of the 1st kind, the coefficientsPoisson fibers and the matrix are respectively equal to: *EB = 7.0 • 104 MPa;EM = 3.5 • 103 MPa; vB = 0.22; vM = 0.35.* The deflection of the plate was measured using dial gaugesup to *0.01 m*m. To measure the strain gage used KF4P1-3-200. Sticker gages carried out according to the instructions on the label AZHV TO 2,782,001. To measure the output signals of strain gages and reporting digitally measuring system used SIIT-3.A mathematical model of calculation is a two-layer planSteen composed of two layers of thick hard transtropich *(1) = h (2) = 1.0 × 10⁻³ m.*
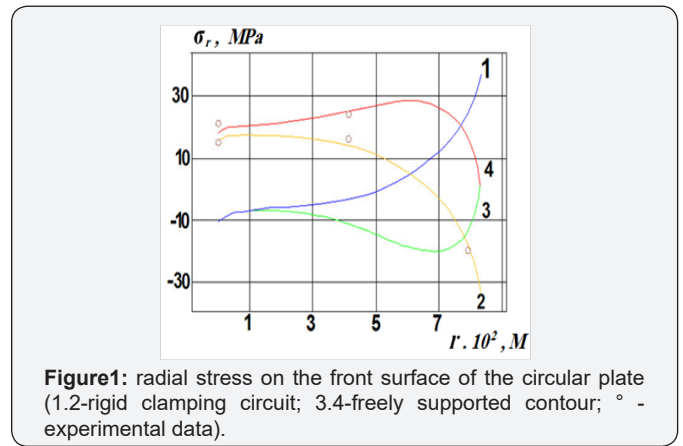

Figure1: radial stress on the front surface of the circular plate (1.2-rigid clamping circuit; 3.4-freely supported contour; ° - experimental data).
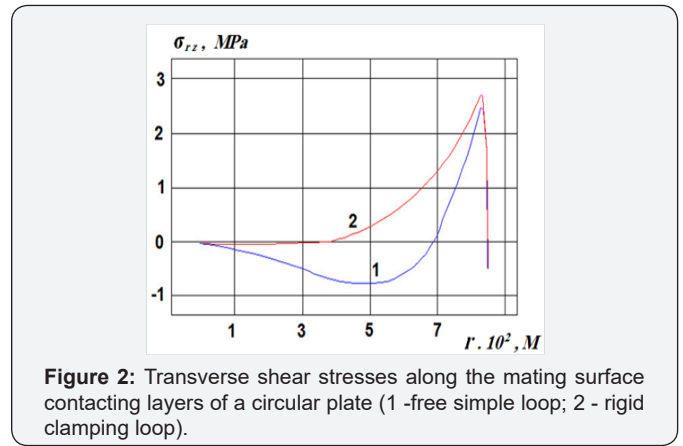

Figure 2: Transverse shear stresses along the mating surface contacting layers of a circular plate (1 -free simple loop; 2 - rigid clamping loop).

It is presumed that the interfacial contact area of the hard layer is very pliable, that is, they may be "slippage" relative to each other.The results of studies of plate bending under the action of uniform pressureq = *0.025 Mpa* are shown in Figure 1-2. The relative error of the theoretical value of the deflection in the center of the plate, when compared with the experimental data was less than *3%: wz = 0.2. 10⁻² m* - A rigid clamping circuit; *ws = 0.2. 10⁻²m* - for a simply supported contour.

Conclusion

According to thenonlinear discrete - structural theory of multi-layer plates and shells the intense deformed case of

anisotropic elements with defects in the structure of the material is investigated. At the conclusion of the equations of balance and geometrical parity the influence of deformations of cross shift and cross-pressure is taken into account. The results of theoretical research are compared with experimental data.

Acknowledgment

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