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An Algebraic Proof and Explanation of the Inner Workings of the Method of Alligation Alternate

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Abstract

The workings of the method of alligation alternate were revealed by an algebraic proof of the method.

Keywords: Alligation; Allegation alternate; Mixture problems; Mass balance equation; Pharmaceutical calculations

Introduction

We have recently developed a method to solve general mathematical mixture problems based on the identification of a set of preserved properties or variables that define a system, construction of equations with these conserved variables at initial and final system states, domain identification of all variables and finally determination of all possible solutions by solving the system of equations [1].

In addition to the general mixture problems, we have used this method to solve mixture composition problems generally encountered in mixing and dilution formulation theories and unraveled many of the limitations of the method of Alligation Alternate. Although, we have shown that, the method of alligation alternate is implicitly using our derived algebraic expressions to reach only one among the infinitude of solutions in three-component mixture composition problems, questions are still remaining. How did the method of alligation alternate came up with assigning the algebraic difference between two or more concentrations to be equal to the volume of the aqueous solution which its concentration is not part of the algebraic subtraction?

Could the practical mathematicians of the early 1650s reached this conclusion empirically by trial and error or did they have in mind a system of equations which they have used to develop the geometrical construct of the method of Alligation Alternate? Available evidence suggests that the method was developed empirically, along with other methods such as the Inverse Rule of Three and the Golden Rule Compound of Five Numbers, from studies related to Euclidean propositions and the practical mathematics of proportions [2,3]. The field of linear algebra in Europe was developed much later and although systems of linear equations started appearing around the same period, alligation alternate always remained in the area of the practical mathematics of proportions.

The method of Alligation Alternate in two-component mixtures is commonly used to determine the volumes of two solutions, and , of known composition, C_1 and C_2, respectively, that can be mixed to make a given volume (V_f) of a solution of an intermediate concentration C_f.

C_1		$C_f - C_2 = V_1$
	C_f	
C_2		$C_1 - C_f = V_2$
		$V_1 + V_2 = V_f$

Figure 1: The alligation alternate method geometric set-up for two-component mixtures.

The rules of the method of alligation alternate for 2-component mixtures are (Figure 1):

1. A three-column diagram is drawn with the concentrations to be mixed in the left column and the desired concentration (C_f) placed in the middle column.

2. The difference, as an absolute value, between the desired concentration and that of each of the aqueous solutions that are to be mixed is calculated and placed in the right column across the concentration that isn't involved in the calculation. The numbers

placed across each concentration are the corresponding *parts* of volume used from each solution.

3. The sum of the parts of volumes in the right column is equal to the *parts* of the desired final mixture (V_f).

Similarly, in three-component mixtures, the method of alligation alternate can be used to determine the volumes of three solutions, V_1, V_2 and V_3 of known composition, C_1, C_2 and C_3, respectively, that can be mixed to make a given volume (V_f) of a solution of a concentration C_f [4]. As mentioned before, the resultant solution (C_f) must be of intermediate concentration. For our demonstration purposes, we have chosen

$$C_1 > C_f > C_2, C_3$$

It is important to note, that volumes of solutions determined by the method of alligation alternate, by subtracting concentrations, are not actual volume quantities. These volumes are always expressed relative to each other (using *parts* as a unit) and these relative numbers or fractions can only yield actual volumes when one of the volumes is assigned an arithmetic value.

The algebraic solution of a two-component mixture composition problem can be reached by solving the following system of equations (eq. 1 and eq. 2), which are derived based on the preservation of the mass of the solute and the preservation of the volume of the solvent in the system, before and after mixing the two solutions.

$$V_1 + V_2 = V_f$$

$$V_1.C_1 + V_2.C_2 = V_f.C_f \qquad \text{eq. 2}$$

Substituting eq. 1 on eq. 2,

$$V_1.C_1 + V_2.C_2 = (V_1 + V_2).C_f$$

$$(C_1 - C_f).V_1 = (C_f - C_2).V_2 \qquad \text{eq. 3}$$

Eq. 3 will always be true when,

$$(C_1 - C_f) = V_2 \qquad \text{eq. 4}$$

$$(C_f - C_2) = V_1 \qquad \text{eq. 5}$$

$$(C_1 - C_2) = V_f \qquad \text{eq. 6}$$

But these are the answers we reached using the diagrammatic approach of the alligation alternate. Regarding the units of our answers, notice that in eq. 3, the units of concentration on both sides of the equation cancel out. V_1 is expressed in terms of V_2, and vice versa, and thus, the volumes are conveniently expressed as fractions or *parts* of total volume in the system.

The algebraic solution for a three-component mixture composition problem can be reached after solving the system of equations below:

$$V_1 + V_2 + V_3 = V_f \qquad \text{eq. 7}$$

$$V_1.C_1 + V_2.C_2 + V_3.C_3 = V_f.C_f \qquad \text{eq. 8}$$

In this case, we have an underdetermined, but consistent, system of linear equations, as the number of unknown variables is higher than the number of available equations. The mathematical interpretation of a consistent system is that the problem has a solution, but not a single one. Instead it has an infinitude of linearly dependent solutions [1]. However, we can still reach one of those solutions by solving the system of equations algebraically.

Substituting from eq. 7 on eq. 8,

$$V_1.C_1 + V_2.C_2 + V_3.C_3 = (V_1 + V_2 + V_3).C_f \qquad \text{eq. 9}$$

Given that $C_1 > C_f > C_2, C_3$ we are going to separate the variables so that on one side we have only concentrations greater than whereas on the other side of the equation we have only concentrations smaller than . This is in accordance with the simple rules of mixing, that is, mixing solutions of given concentrations cannot produce a final mixture of concentration lower than the lowest one or higher than the highest one. For example, mixing a 70 % w/v and a 90 % w/v solution, cannot produce a 40 % w/v mixture.

Taking common factors and separating the variables according to the rules of mixing on opposite sides of the equation:

$$(C_1 - C_f).V_1 = (C_f - C_2).V_2 + (C_f - C_3).V_3 \qquad \text{eq. 10}$$

Eq. 10 will always be true when,

$$(C_1 - C_f) = V_2 = V_3 \qquad \text{eq. 11}$$

$$(C_f - C_2) + (C_f - C_3) = 2C_f - C_2 - C_3 = V_1 \qquad \text{eq. 12}$$

$$2C_1 - C_2 - C_3 = V_f \qquad \text{eq. 13}$$

C_1		$(C_f - C_2) + (C_f - C_3) = 2C_f - C_2 - C_3 = V_1$
C_2	C_f	$(C_1 - C_f) = V_2$
C_3		$(C_1 - C_f) = V_3$
		$V_1 + V_2 + V_3 = V_f$

Figure 2: The alligation alternate method geometric set-up for three-component mixture composition problems $C_1 > C_f > C_2, C_3$.

These are the same answers we reached using the diagrammatic approach of the alligation alternate for three-component mixtures (Figure 2). In addition, using the algebraic equations of the preserved system properties we were able to determine a concentration relationship for the final volume V_f as well. The exact same method can be followed to solve any higher than three-component mixture composition problem, but as with the alligation alternate, this method will provide only one among the infinitude of linearly dependent solutions to the system.

Conclusively, the goal of explaining the inner workings of the

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method of alligation alternate using a system of equations that were developed based on system preserved properties, has been achieved; once and for all.

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